fev 2013 CORRECTION AP. 2de

Ex 1

① $(x-6)^2 - (x^2+36) = x^2 - 12x + 36 - x^2 - 36$
$= -12x$

or $x>0$ d'où $-12x<0$ donc $(x-6)^2 < x^2+36$.

② $\dfrac{2}{3} - \dfrac{20+2x}{29+3x} = \dfrac{2(29+3x)-3(20+2x)}{3(29+3x)}$

$= \dfrac{58+6x-60-6x}{3(29+3x)}$

$= \dfrac{-2}{3(29+3x)}$ or $\dfrac{-2}{3(29+3x)} < 0$ car $x \in \mathbb{R}_+^*$

donc $\dfrac{2}{3} < \dfrac{20+2x}{29+3x}$ $\forall x \in \mathbb{R}_+^*$

③ $\dfrac{n}{n+1} - \dfrac{n+1}{n+2} = \dfrac{n(n+2)-(n+1)(n+1)}{(n+1)(n+2)}$

$= \dfrac{n^2+2n-n^2-2n-1}{(n+1)(n+2)}$

$= \dfrac{-1}{(n+1)(n+2)}$ or $(n+1)(n+2)>0$ car $n \in \mathbb{N}$

d'où $\dfrac{-1}{(n+1)(n+2)} < 0$ donc $\dfrac{n}{n+1} < \dfrac{n+1}{n+2}$

④ $\dfrac{5\sqrt{2}+1}{2\sqrt{2}-1} - \dfrac{4+3\sqrt{2}}{2\sqrt{2}+1} = \dfrac{(5\sqrt{2}+1)(2\sqrt{2}+1)-(4+3\sqrt{2})(2\sqrt{2}-1)}{(2\sqrt{2}-1)(2\sqrt{2}+1)}$

$= \dfrac{10(\sqrt{2})^2+5\sqrt{2}+2\sqrt{2}+1-(8\sqrt{2}-4+6\times 2-3\sqrt{2})}{(2\sqrt{2})^2-1^2}$

$= \dfrac{20+7\sqrt{2}+1-8\sqrt{2}+4-12+3\sqrt{2}}{7}$

$= \dfrac{13+2\sqrt{2}}{7}$

Conclusion :
$\dfrac{13+2\sqrt{2}}{7} > 0$
d'où
$\dfrac{5\sqrt{2}+1}{2\sqrt{2}-1} > \dfrac{4+3\sqrt{2}}{2\sqrt{2}+1}$

Ex 2

① $\sqrt{10}-3>0$ et $\sqrt{19-5\sqrt{10}}>0$ on compare leurs carrés

$(\sqrt{10}-3)^2-(\sqrt{19-5\sqrt{10}})^2 = 10-6\sqrt{10}+9-10+5\sqrt{10}$

$= 9-\sqrt{10} \quad <0$

d'où $\sqrt{10}-3 < \sqrt{19-5\sqrt{10}}$

② $\left(\sqrt{7}+\frac{1}{2}\right)^2-\left(\frac{1}{2}\sqrt{30+\sqrt{7}}\right)^2 = 7+\sqrt{7}+\frac{1}{4}-\frac{1}{4}(30+\sqrt{7})$

$= 7+\frac{1}{4}-\frac{30}{4}+\sqrt{7}-\frac{1}{4}\sqrt{7}$

$= -\frac{1}{4}+\frac{3}{4}\sqrt{7}$

$= \frac{1}{4}(-1+3\sqrt{7}) <0$ car $\sqrt{7}>1$.

or $\sqrt{7}+\frac{1}{2}>0$ et $\frac{1}{2}\sqrt{30+\sqrt{7}}>0$

d'où $\sqrt{7}+\frac{1}{2} < \frac{1}{2}\sqrt{30+\sqrt{7}}$

③ $\left(\sqrt{n^2+4n+3}\right)^2-(n+2)^2 = n^2+4n+3-n^2-4n-4$.

$= -1$

d'où $\left(\sqrt{n^2+4n+3}\right)^2 < (n+2)^2$ or $\sqrt{n^2+4n+3}>0$ et $n+2>0$ (car $n\in\mathbb{N}$)

donc $\sqrt{n^2+4n+3} < n+2$

④ $(\sqrt{7}+1)^2-\left(\sqrt{2\sqrt{7}+7}\right)^2 = 7+2\sqrt{7}+1-2\sqrt{7}-7$

$=1$

d'où $(\sqrt{7}+1)^2 > \left(\sqrt{2\sqrt{7}+7}\right)^2$

or $\begin{cases}\sqrt{7}+1>0\\ \sqrt{2\sqrt{7}+7}>0\end{cases}$ donc $\sqrt{7}+1 > \sqrt{2\sqrt{7}+7}$

5 $(\sqrt{a}+\sqrt{b})^2-(\sqrt{a+b})^2 = a+2\sqrt{ab}+b-a-b$

$= 2\sqrt{ab}$

or $2\sqrt{ab}>0$ d'où $(\sqrt{a}+\sqrt{b})^2 > (\sqrt{a+b})^2$

or $\begin{cases} \sqrt{a}+\sqrt{b}>0 \\ \sqrt{a+b}>0 \end{cases}$ donc $\sqrt{a}+\sqrt{b} > \sqrt{a+b}$

6 $\left(\dfrac{\sqrt{2}+\sqrt{3}}{15}\right)^2 - \left(\dfrac{\sqrt{2+\sqrt{3}}}{9}\right)^2 = \dfrac{2+2\sqrt{6}+3}{225} - \dfrac{2+\sqrt{3}}{81}$

$= \dfrac{(5+2\sqrt{6})\times 81 - 25(2+\sqrt{3})}{2025}$

car $\left(\begin{array}{l} 2025 = 3^4\times 5^2 \\ 15^2 = 3^2\times 5^2 \\ 9^2 = 3^4 \end{array}\right)$

$= \dfrac{385+137\sqrt{3}}{2025}$

or $\dfrac{385+137\sqrt{3}}{2025} > 0$ d'où $\left(\dfrac{\sqrt{2}+\sqrt{3}}{15}\right)^2 > \left(\dfrac{\sqrt{2+\sqrt{3}}}{9}\right)^2$

or $\begin{cases} \dfrac{\sqrt{2}+\sqrt{3}}{15} > 0 \\ \dfrac{\sqrt{2+\sqrt{3}}}{9} > 0 \end{cases}$ donc $\dfrac{\sqrt{2}+\sqrt{3}}{15} > \dfrac{\sqrt{2+\sqrt{3}}}{9}$