

Ejercicio 26:

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$$S_\lambda \equiv \begin{cases} -6y - 9z - 9x\lambda = 0 \\ 2x - 2z - 2y\lambda = 0 \\ 2y - 2x + 2z = -2\lambda \end{cases}$$

$$S_\lambda = \begin{pmatrix} -9\lambda & -6 & -9 \\ 2 & -2\lambda & -2 \\ -2 & 2 & 2 \end{pmatrix} \quad \checkmark$$

$$\det(S) = 36\lambda^2 - 24 - 36 + 36\lambda + 24 - 36\lambda = 36\lambda^2 - 36 = 36(\lambda^2 - 1) \quad \checkmark$$

2º caso Para $\lambda \notin \{1, -1\}$: $\det(S) \neq 0$ *no tiene mas un sistema compatible de determinación (Cramer)*

$$S_x = \begin{pmatrix} 0 & -6 & -9 \\ 0 & -2\lambda & -2 \\ -2\lambda & 2 & 2 \end{pmatrix} \quad \lambda^2 - 1 = 0 \Rightarrow \lambda = 1 \vee \lambda = -1 \quad \checkmark$$

$$\det(S_x) = -24\lambda + 36\lambda^2$$

$$S_y = \begin{pmatrix} -9\lambda & 0 & -9 \\ 2 & 0 & -2 \\ -2 & -2\lambda & 2 \end{pmatrix}$$

$$\det(S_y) = 36\lambda + 36\lambda^2$$

$$S_z = \begin{pmatrix} -9\lambda & -6 & 0 \\ 2 & -2\lambda & 0 \\ -2 & 2 & -2\lambda \end{pmatrix}$$

$$\det(S_z) = -36\lambda^3 - 24\lambda$$

$$\begin{cases} x = \frac{36\lambda^2 - 24\lambda}{36\lambda^2 - 36} = \frac{12(3\lambda^2 - 2\lambda)}{12(3\lambda^2 - 3)} = \frac{3\lambda^2 - 2\lambda}{3\lambda^2 - 3} \quad \checkmark \\ y = \frac{36\lambda + 36\lambda^2}{36\lambda^2 - 36} = \frac{36(\lambda^2 + \lambda)}{36(\lambda^2 - 1)} = \frac{\lambda^2 + \lambda}{\lambda^2 - 1} \quad \left(= \frac{3\lambda^2 + 3\lambda}{3(\lambda^2 - 1)} \right) \\ z = \frac{-36\lambda^3 - 24\lambda}{36\lambda^2 - 36} = \frac{12(-3\lambda^3 - 2\lambda)}{12(3\lambda^2 - 3)} = \frac{-3\lambda^3 - 2\lambda}{3\lambda^2 - 3} \quad \checkmark \end{cases}$$

Segundo caso

Para $\lambda = 1$:

$$S_{\pm} \equiv \begin{cases} -6y - 9z - 9x = 0 \\ 2x - 2z - 2y = 0 \\ 2y - 2x + 2z = -2 \end{cases} \Leftrightarrow \begin{cases} 2x - 2y - 2z = 0 \\ -2x + 2y + 2z = -2 \\ -9x - 6y - 9z = 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} 2x - 2y - 2z = 0 \\ 0 = -2 \quad (L_2 + L_1) \end{cases} \quad \text{sistema incompatible}$$

Tercer caso

Para $\lambda = -1$

$$S_{-1} \equiv \begin{cases} -6y - 9z + 9x = 0 \\ 2x - 2z + 2y = 0 \\ 2y - 2x + 2z = 2 \end{cases} \Leftrightarrow \begin{cases} 2x + 2y - 2z = 0 \\ -2x + 2y + 2z = 2 \\ 9x - 6y - 9z = 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} 2x + 2y - 2z = 0 \\ 4y = 2 \\ -30y = 0 \end{cases} \begin{matrix} (L_2 + L_1) \\ (2L_3 - 9L_1) \end{matrix} \Leftrightarrow \begin{cases} 2x + 2y - 2z = 0 \\ y = \frac{1}{2} \\ y = 0 \end{cases} \checkmark$$

sistema incompatible

$$\text{Para } \lambda \notin \{-1; 1\} \quad S = \left\{ \left(\frac{3\lambda^2 - 2\lambda}{3\lambda^2 - 3}; \frac{\lambda^2 + \lambda}{\lambda^2 - 1}; \frac{-3\lambda^3 - 2\lambda}{3\lambda^2 - 3} \right) \lambda \in \mathbb{R} \right\}$$

$$\text{Para } \lambda \in \{-1; 1\} \quad S = \emptyset$$